COMMENTS ON THE ARTICLES "CALCULATING THE FLOW OF A TWO-PHASE STREAM IN AN AXISYMMETRIC SUPERSONIC NOZZLE"* AND " THE FLOW OF A GAS–LIQUID MIXTURE IN A SHAPED NOZZLE, WITH A CONSTANT PHASE VELOCITY DIFFERENCE" †

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In the article "Calculation of the flow of a two-phase stream in an axisymmetric supersonic nozzle" by I. M. Kapura et al. [Inzhen.-Fiz. Zh., 13, No. 5 (1967)] and in the article "The flow of a gas–liquid mixture in a shaped nozzle, with a constant phase velocity difference" by V. G. Selevanov and S. D. Frolov [Inzhen.-Fiz. Zh., 12, No. 5 (1967)] the problems of the one-dimensional flow of two-phase media in nozzles are examined.

A system of differential equations is presented in the first article to describe the flow of a two-phase stream along an axisymmetric supersonic nozzle, and the computer calculation results are given. In the derivation of these equations certain assumptions are made, and these are similar to such earlier works as [1-4]. Moreover, the authors assume that there exists no transfer of heat between the gas and the particles, thus markedly restricting the area in which the calculation results can be applied, since this assumption is valid in the case of low temperatures and large particles. However, in solving problems of this type on an electronic digital computer there is no need for any such assumptions with regard to an absence of heat transfer between the gas and the particles, since the addition of the heat-transfer equations to the system of differential equations introduces no noticeable computational difficulties (see, for example, [1-3]).

With regard to the system of equations brought to a form convenient for numerical integration – system (3) – it should be pointed out that the equation of motion for the particles is normally integrated in the following form (for example, [3, 9]):

$$\frac{dw_g}{dx} = \nu\left(\frac{w_g}{w_s} - 1\right). \tag{1}$$

As regards the second equation of system (3), this equation is incorrect. In the authors' notation this equation should have the following form:

$$\frac{dw_g}{dx} = \frac{1}{M^2 - 1}\left[\frac{w_g}{F}\frac{dF}{dx} - \frac{g_s}{g_g}\frac{w_g}{R_g T_g}\left(w_g - \frac{\gamma_g - 1}{\gamma_g} w_s\right)\frac{dw_s}{dx}\right]. \tag{2}$$

The equation used by the authors is derived from the above equation, if we arbitrarily assume that $w_g = w_s$ in the parentheses.‡

Equations analogous to Eq. (2) are derived if we assume $dT_s = 0$ in relationship (10) of reference [3] or in Eq. (8) of reference [4], which the authors cite in a reference section. Consequently, equally

*I. M. Karpura et al., Inzhen.-Fiz. Zh., 13, No. 5 (1967).
†V. G. Selivanov and S. D. Frolov, Inzhen.-Fiz. Zh., 12, No. 5 (1967).
‡If the authors, for the sake of simplification, deliberately introduced the condition $w_g = w_s$ (whose purpose is entirely unjustified), special mention of this fact should have been made.

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unreliable are the consequences of this equation, in particular, the author's condition which should be satisfied when $M_g = 1$; the correct form of this condition is as follows:

$$\frac{w_g}{F}\frac{dF}{dx} = \frac{g_s}{g_g}\frac{w_g}{R_g T_s}\left(w_g - \frac{\gamma_g - 1}{\gamma_g} w_g\right)\frac{dw_g}{dx}. \tag{3}$$

Thus all of the numerical results of the calculations are incorrect, although qualitatively they are analogous to the results of the earlier calculations [1-3].

The authors say absolutely nothing about the fundamental difficulty which arises in the solution of the immediate problem of determining the parameters of a two-phase mixture in a nozzle of fixed geometry, namely: the difficulty in numerical integration of passing the point $M_g = 1$, since the singularity $(dw_g/dx) = (0/0)$ arises in this case. A possible means of eliminating this difficulty has been presented in reference [3]; here we also find the method and an example for a numerical calculation of the flow of a gas with particles in a nozzle exhibiting a fixed geometry. However, the passage of the point $M_g = 1$ is not covered by the authors in their paper.

In the literature on two-phase flows in nozzles, the specific impulse is defined as momentum referred to the total pre-second flow rate of the gas and particles. However, the authors refer the momentum of the two-phase jet exclusively to the flow rate of the gas phase, failing to make any special mention of this fact. Naturally, they achieve results opposite to those of [2, 4] where it is concluded that the weight fraction of the particles exerts influence on the specific impulse, and thus totally unnecessary questions are raised in a rapid reading of the article.

Thus there is absolutely nothing new in this article relative to the earlier and more thorough papers dealing with this question; an unhappy assumption and the presence of errors makes the results of the numerical calculations valueless.

In the Selivanov and Frolov article "...the combined flow of the gas and liquid drops is examined in a one-dimensional approximation for flow in a shaped nozzle, given a constant velocity difference between the phases," i.e., they are solving the inverse problem of constructing a nozzle where the conditions are imposed on the velocity. As follows from their equations of the flow rate for the gas and the particles, the authors seek to take into consideration the spatial volume occupied by the particles.

However, it follows from an examination of the momentum and energy equations of the authors that they assume that the area occupied by the particles is small in comparison with that occupied by the gas, i.e., here they actually fail to take into consideration the space occupied by the particles. A problem similar to that examined by the authors had been treated under more general conditions by Hassan [7], and an analytical solution was derived there. The case of continuous velocity lag had been dealt with by Kliegel [4] as early as 1960. To obtain a solution in elementary functions, the authors have to assume that there exists no transfer of heat between the gas and the particles, and we have discussed the validity of this assumption earlier. However, even after this, their solution is cumbersome and inconvenient for purposes of calculation and its physical significance is not self-evident. At the same time, Kliegel demonstrated that the equations which describe the one-dimensional flow of a gas mixture with a continuous velocity and temperature lag are identical to the equations describing the one-dimensional flow of an ideal gas with other values for γ_g and R_g, i.e., to determine the flow parameters for a two-phase medium we can use the gas-dynamic tables, and the calculations are thus essentially reduced to the minimum. The authors thus repeated the solution of a problem that has already been solved, and they did this without any improvement, demonstrating their inadequate familiarity with the literature on this subject.

In conclusion, it should be pointed out that both the domestic and foreign literature is concerned with considerably more complex problems relating to the flow of two-phase mixtures in nozzles. A detailed examination of the problem of the flow of a two-phase mixture, with consideration of the space occupied by the particles, can be found in [6, 7]; calculations have been carried out for two-dimensional flows of a gas with particles by the method of characteristics [8, 9]; in one-dimensional approximation, a solution has been derived for the problem of constructing optimum nozzle contours for gas flows with lagging particles [10-12]; calculations have been carried out for flows in nozzles, with consideration of particle coagulation [13], etc.

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REPLY TO STARKOV'S COMMENTS ON THE ARTICLE: "CALCULATING THE FLOW OF A TWO-PHASE STREAM IN AN AXISYMMETRIC SUPERSONIC NOZZLE"

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In the derivation of the system of equations, Starkov objects to our assumption that no heat transfer takes place between phases. We resorted to this assumption not in order to reduce computer machine time, as concluded by Starkov, but to ascertain the effect of an exclusively mechanical action (the friction of the gas against the particles) on the progress of a two-phase flow in a nozzle. Consideration of the effect of heat transfer clouds the issue. In our paper we have presented a system of equations both with and without consideration of heat transfer. Here, however, on the basis of computer calculations, it is demonstrated that consideration of heat transfer leads to but a slight change in the adiabatic efficiency of the discharge process and in the specific impulse, as referred to the gas phase.

The equation of motion for a solid particle, recommended by Starkov, is a special case of the one given in our paper. The coefficient c_x in our equation is a function of $\mathrm{Re}_{\mathrm{rel}}$, where

$$\mathrm{Re}_{\mathrm{rel}} = \frac{(w_g - w_s)\, d_g \gamma_g}{\mu_g g}.$$

Here we assumed that

$$\text{for} \quad \mathrm{Re}_{\mathrm{rel}} < 5.8 \quad c_x = \frac{24}{\mathrm{Re}_{\mathrm{rel}}},$$

$$\text{for} \quad 5.8 \leqslant \mathrm{Re}_{\mathrm{rel}} \leqslant 730 \quad c_x = \frac{13}{\sqrt{\mathrm{Re}_{\mathrm{rel}}}},$$

$$\text{for} \quad \mathrm{Re}_{\mathrm{rel}} > 730 \quad c_x = 0.48.$$

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